Evaluating improper response test data by using superposition of line source approximation

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ABSTRACT

This paper presents a new evaluation method for the Geothermal Response Test (GeRT, TRT), able to cope with improper response test data, e.g. as a result of instable power. TRT meanwhile is a standard method for determining the thermal conductivity of the underground and the borehole resistance. The test rig is connected to the BHE and water (or brine) is circulated in the system. By injecting thermal energy (heating) the fluid temperatures rise with the time. The slope of the temperature curve is correlated to the thermal conductivity.

Temperatures, flow rate and heating load are measured. By evaluating the data of the temperature development, the thermal conductivity of the underground can be calculated. For this evaluation several methods and mathematical models are available, the most common being the analytical model of Kelvin's line source theory, followed by cylinder source and numerical simulation methods (e.g. with finite elements). Whereas the latter are able to cope with different power levels or fluctuations in power, the analytical models assume that heating load stays constant over the whole testing period. Fluctuations of the power supply may lead to misinterpretation and instable results (detectable with the stepwise / sequential evaluation).

By using the approach of ESKILSON (1987) for the superposition of the line source approximation all kind of power fluctuations and variations can be handled. With the method the temperature development is calculated using the different heating loads for each time step. The thermal conductivity and borehole resistance are varied within predetermined limits and the resulting temperature curve is compared with the measured temperatures. The parameters of the best fit curve are regarded as the result.

A computer program was developed to evaluate GeRT data using the superposition method. Several calculations were performed using test data with showing a stable result in stepwise / sequential evaluation. By comparison of 21 tests, the deviation between standard line source method and superposition method is less than 3%.

The superposition method is an easy to use and fast method to find adequate results for test runs with improper power supply or disturbance by environmental influences (solar radiation etc.).

1. INTRODUCTION

The conductive heat transport in an infinite homogeneous medium around an infinite line heat source can be described with the line source equation developed by KELVIN in the 19th century (THOMSON, 1884). The equation is shown below as described by INGERSOLL and PLASS (1948):

$$\Delta T_{(r,t)} = \frac{q}{4\pi\lambda H}\int_{p}^{\infty}\frac{e^{-\beta^2}}{\beta}d\beta \qquad (1)$$

$$p = \frac{r}{2\sqrt{\alpha\, t}}$$

- λ = thermal conductivity [W/m*K]
- α = thermal diffusivity [m²/s]
- H = length of tube [m]
- q = temperature initiation rate [W]
- r = radius [m]
- β = integration constant
- t = time from the test beginning [s]
- ΔT = temperature difference [K]
- c_V = vol. heat capacity [J/K*m³]

Usually for evaluation of response test data the approximation of MOGENSEN (1983) explemented with the term considering thermal borehole resistance after GEHLIN (1998) is used:

$$T_f = \frac{Q}{4\pi\lambda H}\ln(t) + \left[\frac{Q}{H}\left(\frac{1}{4\pi\lambda}\left(\ln\left(\frac{4\alpha}{r_b^2}\right) - \gamma\right) - R_b\right) + T_0\right] \qquad (2)$$

- Q = Heating output [W]
- H = Length of BHE [m]
- T_0 = Undisturbed ground temperature [°C]
- λ = Thermal conductivity [W/(m K)]
- α = Thermal diffusivity [m²/s]
- r_0 = Radius of borehole [m]
- t = Time [sec]
- T_f = Fluid temperature at time t [°C]
- γ = Euler's constant (0,5772)

To describe the linear behavior of the (mean) fluid temperatures [T_f] in an BHE against logarithmic time scale [ln(t)] the equation can be simplified (GEHLIN, 1998) to:

$$T_f = k \ \ln(t) + m \qquad (3)$$

By transposing the first term of equation (2) and with known heat injection rate [Q] and length of the BHE [H] the thermal conductivity [λ] is defined by the gradient of the temperatures plotted in semi log scale against time:

$$\lambda = \frac{Q}{4\,\pi\,k\,H} \qquad (4)$$

The method leads to suitable results only if the heat injection rate is constant. Typically the mean value over the test period is used. Smaller fluctuations are compensated statistically over the test duration within certain limits.

The result will be not within confidence if heat injection rate fluctuates strongly, e.g. because of high fluctuations in the power grid or changes of the air temperature (very cold during night in winter times or high solar radiation by day during summer). Generally, fluctuations with high amplitude, but being statistically balanced, are less disturbing than drifting fluctuations with low amplitude.

The impact of influencing factors can be elucidated by using the stepwise (sequential) evaluation (GEHLIN, 1998, SANNER et al. 2007). With this method a forward stepwise evaluation of the recorded data with a fixed start time and variable end time is performed. The resulting thermal conductivity for each time-span can be calculated and plotted over time.

Usually in the first part of such a curve the thermal conductivity swings up and down, converging to a steady value and a horizontal curve in the case of a perfect test. If the curve continues swinging up and down, the test time has to be extended. If the curve continues to rise, a high groundwater flow exists.

The example in figure 1 shows the data (temperatures, heating load, stepwise evaluation) of a quite stable test with low fluctuations.

The example in figure 2 shows a heavily disturbed test. Strong power fluctuations lead to inconsistent temperature development. The stepwise evaluation does not stabilize. The test data cannot be evaluated with standard methods.

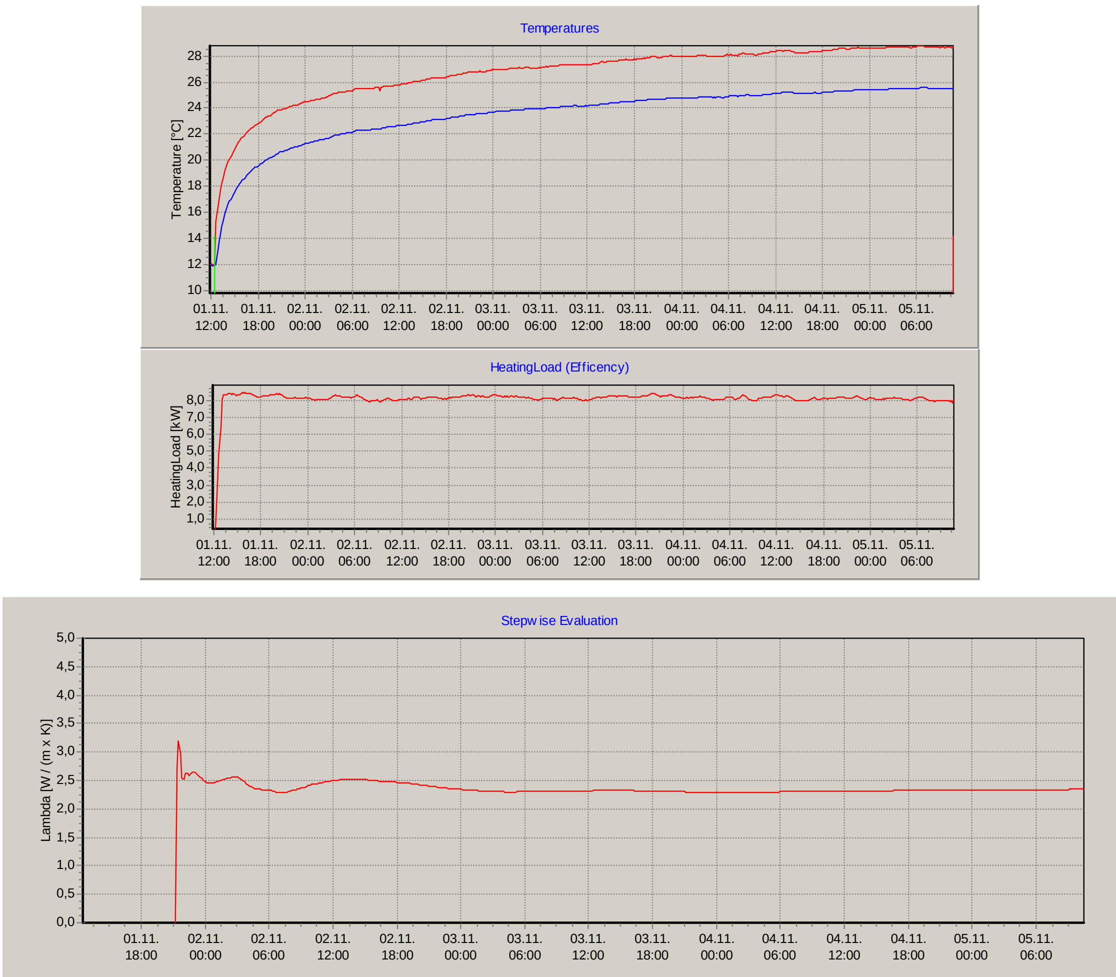

Figure 1: Data of stable TRT (above) and stepwise evaluation (below)

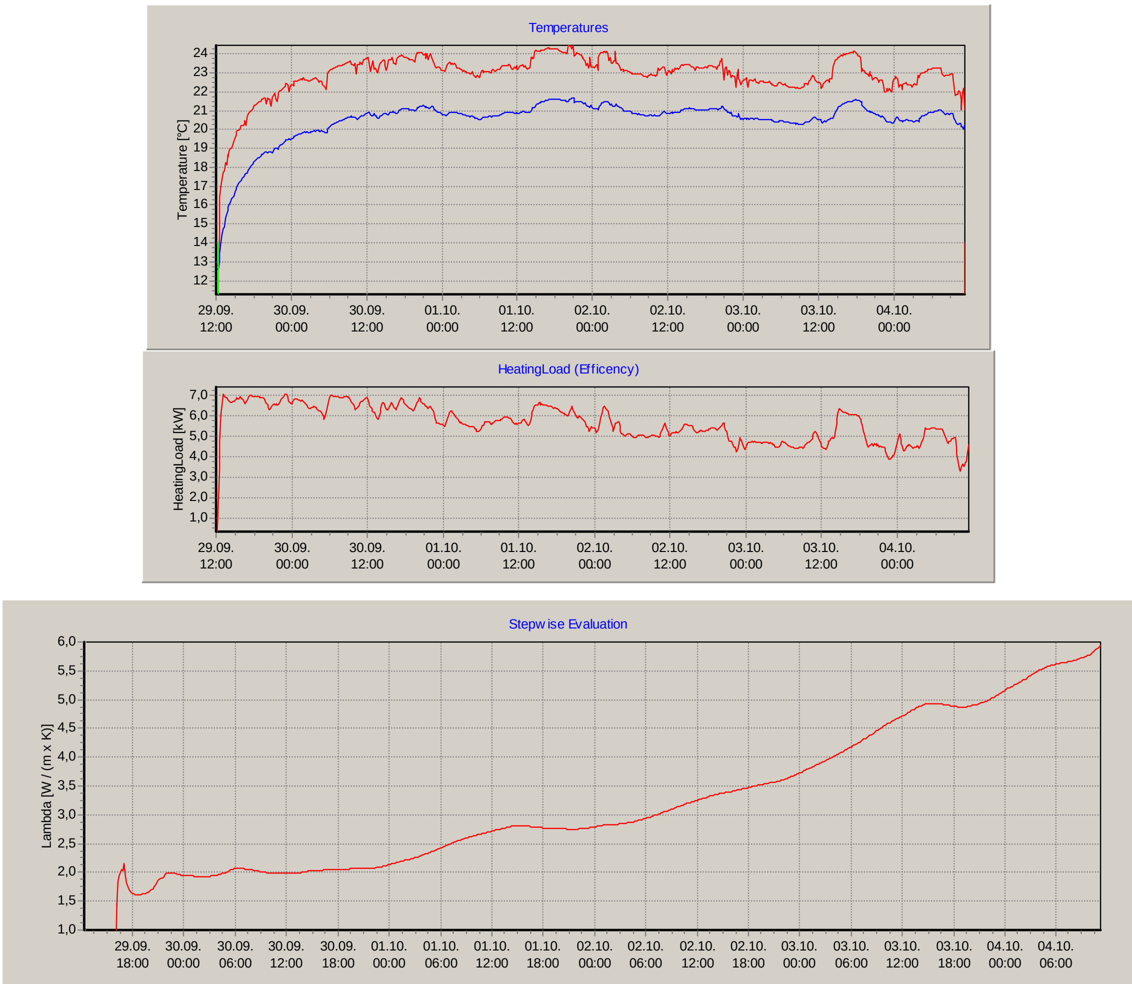

Figure 2: Data of instable TRT (above) and stepwise evaluation (below)

To get a result from such test runs as shown in figure 2, the application of numerical heat transport simulation models is necessary. Usually this kind of models is very complex and time intensive.

2. SUPERPOSITION METHOD

A more comfortable way to deal with inconsistent data is the superposition of line source approximation. The method was described by ESKILSON (1987) for calculation of temperature development at the borehole wall with changing heating loads. The simulation software EED (Earth Energy Designer) is based on this method as well (HELLSTRÖM et al. 1997).

If the term for borehole resistance from equation 2 is added, the method can be used for evaluating TRT data as well. The temperature development in the BHE under changing heat extraction and/or injection conditions can be described by single current pulses which continue indefinitely and superpose each other:

$$T_{f_n} = T_0 - \sum_{n=1}^{N} \frac{Q_n - Q_{n-1}}{H}\left(R_b + \left(\frac{1}{4\pi\lambda}\right)\cdot\left(\ln\left(\frac{4\alpha}{r_b^2}\right) + \ln(t-t_n) - \gamma\right)\right) \quad (5)$$

The principle is shown in the figures below. Fig. 3 shows the temperature development of a constant heat extraction of 10 W/m for the duration of 60 hours. Five hours after the first pulse had started, the extraction rate is increased to 15 W/m and beginning at hour 6, a second pulse with 5 W/m (fig. 4) is added. The resulting temperature development is shown in fig. 5.

Based on equation 5, a computer program was developed that is able to simulate a temperature curve using the heating load data from a TRT. The program compares the calculated values with the measured temperatures by determining the standard deviation between the two curves. Under variation of thermal conductivity (λ) and borehole resistance (r_b) a search algorithm finds a simulated curve which has the best fit to the measurements. The parameters λ and r_b of this best fit curve (with the lowest standard deviation to the measurements) are considered as the result.

3. VALIDATION

The test run shown in fig. 2 was simulated with the superposition method. Fig. 6 shows the result and the measured temperatures. In addition the result of a 3D finite element (FEM) simulation (performed with FeFlow 6) of this test run is displayed.

FEM-method and superposition method lead to the same result of thermal conductivity (λ = 2,4 W/(m·K)) and borehole resistance (r_b = 0,07 (m·K)/W).

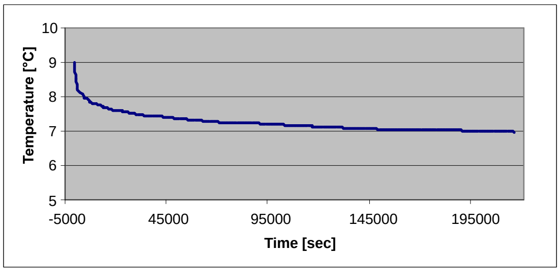

Figure 3: Pulse 1: 10 W/m for 60 h (hour 1 to hour 60)

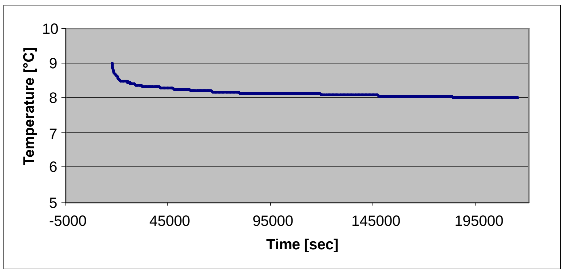

Figure 4: Pulse 2: 5 W/m for 54 h (hour 6 to hour 60)

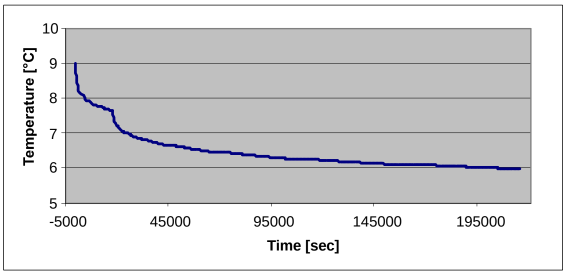

Figure 5: Sum pulse 1 + pulse 2

For further validation, 21 test runs have been evaluated with the superposition method, and results were compared with those of the standard method. The tests are characterized by low power fluctuations and very stable stepwise evaluation. Therefore both methods should show the same result. Another five test runs with high power fluctuations had been compared to the results of FEM simulation. Table 1 shows the results and the standard deviation between different methods. Fig. 7 shows the results in a λ/λ diagram.

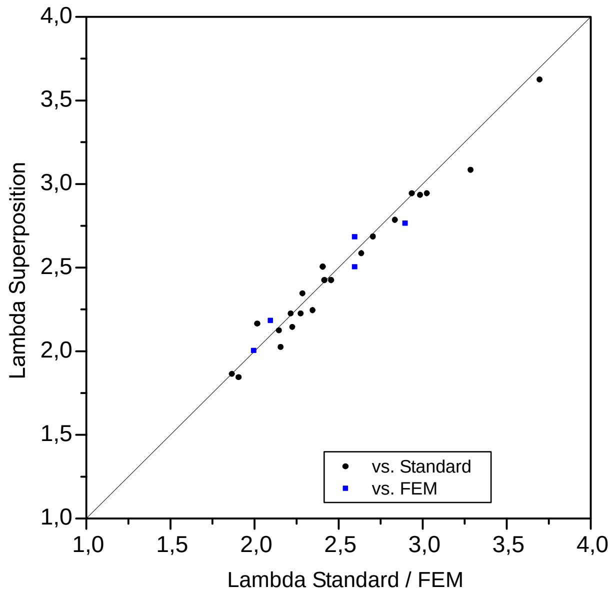

Figure 7: λ/λ diagram Superposition against Standard/FEM

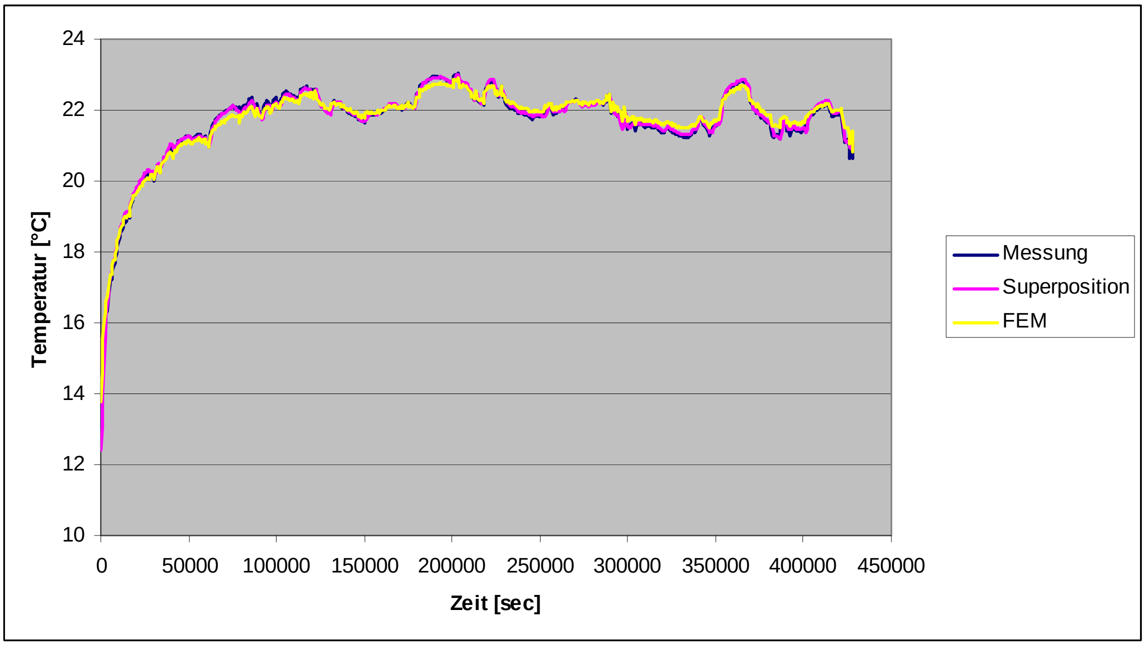

Figure 6: Measured temperatures, temperatures simulated with superposition method, and temperatures simulated with FEM of an instable test run

Table 1: Comparison of evaluation results from standard method / FEM to superposition method

GeRT No.	Thermal conductivity Standard/FEM [W/(m·K)]	Thermal conductivity superposition [W/(m·K)]	Difference [W/(m·K)]	Standard deviation [-]	Deviation %
1	3,70	3,62	0,08	0,060	2,2
2	2,64	2,58	0,06	0,038	2,3
3	3,03	2,94	0,09	0,105	3,0
4	2,15	2,12	0,03	0,077	1,4
5	2,22	2,22	0,00	0,099	0,0
6	2,02	2,16	0,14	0,149	6,9
7	2,42	2,42	0,00	0,140	0,0
8	2,99	2,93	0,06	0,078	2,0
9	2,28	2,22	0,06	0,090	2,6
10	2,35	2,24	0,11	0,088	4,7
11	2,29	2,34	0,05	0,078	2,2
12	2,23	2,14	0,09	0,169	4,0
13	2,71	2,68	0,03	0,097	1,1
14	1,91	1,84	0,07	0,092	3,7
15	2,46	2,42	0,04	0,060	1,6
16	2,41	2,50	0,09	0,273	3,7
17	2,84	2,78	0,06	0,070	2,1
18	1,87	1,86	0,01	0,102	0,5
19	2,94	2,94	0,00	0,059	0,0
20	2,16	2,02	0,14	0,107	6,5
21	3,29	3,08	0,21	0,091	6,4
FEM 1	2,90	2,76	0,14		4,8
FEM 2	2,60	2,68	0,08		3,1
FEM 3	2,60	2,50	0,10		3,8
FEM 4	2,00	2,00	0,00		0,0
FEM 5	2,10	2,18	0,08		3,8
Average			**0,070**	**0,101**	**2,8**

4. CONCLUSIONS

Finally the comparison shows that the superposition method gives almost the same result as the standard method in case of stable test runs and in cases of unstable test runs evaluated with FEM method as well.

The average deviation between superposition and standard/FEM is about 2,8%. Therefore the superposition method can be considered as an adequate method to evaluate proper and improper test data

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